An Evolutionary model of innovation, imitation and competition with heterogeneous agents

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Outline

1. Introduction
2. The benchmark model
3. An extended model
4. Conclusions and further research
Purpose of this study

Two alternative ways to an innovative product or process are R&D investment and imitation of others’ innovation. Model the dynamics of innovation and imitation in a market with many firms and monopolistic competition.

- What role does heterogeneity of firms and their externalities play in the dynamics of innovation?
- What is the impact of factors like appropriability of knowledge on such dynamics?
- Are there homogenous equilibria where everybody innovate or imitate?
- What is the long run dynamics beyond stable equilibrium?

Benchmark model: innovation as costs reduction
Extended model: effects on demand (product differentiation).
Assumptions

An industry with $N$ firms producing a slightly differentiated good. Fraction $n^{INN}$ innovate, fraction $n^{IM}$ imitate.

Products are homogeneous and symmetric wrt costs but not perfect substitutes.

Demand is linear and homogenous, $D(p) = a - bp$

Supply by each type is linear (quadratic costs), total supply is heterogeneous and equal to a weighted average of innovators and imitators, $S(p) = n^{INN} S^{INN}(p) + n^{IM} S^{IM}(p)$

Market equilibrium in period $t$: cobweb model and perfect foresight of price in next period (rational expectations)

$$a - dp_t = n^{INN}_{t-1} S^{INN}_t p_t + n^{IM}_{t-1} S^{IM}_t p_t$$ (1)
The theoretical framework

How do agents choose whether to innovate or to imitate? Adopt the discrete choice model of Brock and Hommes (1997): choice probabilities are given by a logit distribution

\[
\begin{align*}
n_t^{INN} &= \frac{e^{\beta \pi_t^{INN}}}{e^{\beta \pi_t^{INN}} + e^{\beta \pi_t^{IM}}} = \frac{1}{1 + e^{-\beta \Delta \pi_t}}, \quad n_t^{IM} = 1 - n_t^{INN} \\
\end{align*}
\]

(2)

The difference of profits \(\Delta \pi_t\) is a fitness measure. Profits read

\[
\begin{align*}
\pi_t^{INN} &= p_t S^{INN}(p_t) - c(S^{INN}(p_t)) - C = \frac{1}{2} s^{INN} p_t^2 - C \\
\pi_t^{IM} &= p_t S^{IM}(p_t) - c(S^{IM}(p_t)) = \frac{1}{2} s^{IM} p_t^2 \\
\end{align*}
\]

(3) (4)

\(\beta\) is the intensity of choice, \(c(\cdot)\) the production costs function and \(C\) the \(R&D\) fixed costs (constant in time).
Innovators vs imitators

Modelling imitation, two basic ideas:

1. imitation works better the more innovators are around.
2. imitators enjoy innovation up to a replicability factor.

\[ s^{INN} = se^{bC}, \quad s^{IM} = \mu n^{INN} se^{bC} \] (5)

\( C \) is the fixed cost of R&D, \( b \) the benefits rate of innovation investment.

The factor \( \mu n^{INN} \) represents the positive dynamic externality of innovators on imitators.

Parameter \( \mu \in [0, 1] \) is a static level of replicability and is linked to the appropriability of knowledge.

Substitute into eq. (1) and obtain market equilibrium at time \( t \)

\[ a - dp_t = n_{t-1}^{INN} se^{bC} p_t + n_{t-1}^{IM} \mu n_{t-1}^{INN} se^{bC} p_t \] (6)
Theoretical setting

A dynamical system

The model describes a uni-dimensional dynamic system where each of $n_{t}^{INN}$, $p_{t}$, $\pi_{t}^{INN}$ and $\pi_{t}^{IM}$ can work as state variable. The price at time $t$ is

$$p_{t} = \frac{a}{d + s n_{t-1}^{INN} e^{bC} [1 + \mu (1 - n_{t-1}^{INN})]} \quad (7)$$

Substituting into the expression of innovators’ fraction (2) one obtains an uni-dimensional map $n_{t}^{INN} = f(n_{t-1}^{INN})$ with

$$f(x) = \frac{1}{1 + e^{\beta \left[ \frac{1}{2} s e^{bC} a^2 \left\{ d + s e^{bC} x [1 + \mu (1-x)] \right\}^2 + C \right]}} \quad (8)$$
An example

Very low innovation benefits, $b << 1$, and perfectly replicable innovation $\mu = 1$. Two examples for two different values of innovation costs: $C = 0.4$ (stable) and $C = 4$ (unstable).

Other parameters are $a = 4$, $d = 1$, $s = 2$ and $\beta = 1$. 
The system may converge to a heterogeneous stable equilibrium,

Time series of $n^{INN}_t$

Graphical analysis

Here $C = 1.1$ ($b = 1$, $\mu = 1$, $\beta = 2$).
The system may converge to a heterogeneous stable equilibrium,

**Time series of** $n_t^{INN}$

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or converge to a stable 2-cycle:

**Time series of $n_t^{INN}$**

![Graph of time series of $n_t^{INN}$](image)

**Graphical analysis**

![Graphical analysis](image)

Here $C = 1.2$ ($b = 1$, $\mu = 1$, $\beta = 2$).
or converge to a stable 2-cycle:

Time series of $n_t^{INN}$

Graphical analysis

Here $C = 1.2$ ($b = 1$, $\mu = 1$, $\beta = 2$).
Bifurcation diagrams: the intensity of choice $\beta$

A BD tells stability analysis and qualitative behaviour.

Here a period doubling bifurcation. The industry is stable only for low intensity of choice.

\[ n_t^{INN} \text{ wrt } \beta \]

\[ p_t \text{ wrt } \beta \]

\[ (\mu = 0.7, \ s = 2, \ C = 1, \ b = 0.2, \ a = 4 \text{ and } d = 1). \]
Bifurcation diagrams: the intensity of choice $\beta$

A BD tells stability analysis and qualitative behaviour.

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B.D. of $n_t^{INN}$ wrt $\beta$

B.D. of $p_t$ wrt $\beta$

$(\mu = 0.7, s = 2, C = 1, b = 0.2, a = 4$ and $d = 1)$. 
Intensity of choice $\beta$ (II)

Period doubling and period halving bifurcations. With lower replicability $\mu$ and lower $s$ the industry returns to be stable when the intensity of choice becomes large enough.

$B.D. \text{ of } n_t^{INN} \text{ wrt } \beta$

$B.D. \text{ of } p_t \text{ wrt } \beta$

($\mu = 0.4$, $s = 1$, $C = 1$, $b = 0.2$, $a = 4$ and $d = 1$)
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Period doubling and period halving bifurcations. With lower replicability $\mu$ and lower $s$ the industry returns to be stable when the intensity of choice becomes large enough.

B.D. of $n_t^{INN}$ wrt $\beta$

B.D. of $p_t$ wrt $\beta$

$(\mu = 0.4, s = 1, C = 1, b = 0.2, a = 4 \text{ and } d = 1)$
The replicability (or imitators efficiency) $\mu$

Up to a level the system is stable and replicability hurts innovation. Above the industry becomes unstable: more and more firms shift behaviour periodically. For $\mu = 1$ almost all firms shift every period.

$(\beta = 2, s = 2, b = 0.2, C = 1, a = 4$ and $d = 1)$
Innovation benefits rate $b$

Two values of the intensity, $\beta = 1.2$ (blue) and $\beta = 2$ (red). When innovation benefits grow larger, more firms imitate. Threshold value of benefits below which the industry is unstable. Such a threshold is larger for higher intensity of choice.

$(s = 2, C = 1, \mu = 0.7, a = 4 \text{ and } d = 1)$
Comments

- Costly agents survive in a deterministic environment, thanks to the opposing forces of innovation externality.
- Plenty of stable equilibria where both agents are present.
- With $C = 0$ and $\mu = 1$ the population does not split equally, again because of innovation externality.
- Role of replicability not univocal. As $\mu$ grows, innovators decrease up to a point, when the system becomes unstable.
- A larger intensity of choice makes the industry unstable, but not always.
- Instability boils down to period 2 oscillations: still economic coherence. Chaos is avoided.
Innovation that also affects demand

Not just costs reduction but also product differentiation: prod. different. ⇒ substitutability ↓ ⇒ demand shifts outwards.

Define a dynamic product substitutability $\xi_t$ and assume the more innovators are around, the lower substitutability:

$$\xi_t = \xi(1 - n_t^{INN}) \quad (9)$$

Demand shifts outwards (Lin and Saggi, 2002):

$$D_t(p_t) = a - dp_t \over 1 + \sigma(1 - n_t^{INN}) \quad (10)$$

Generalized substitutability $\sigma = \xi N$: a measure of competition.

The market equilibrium in period $t$ becomes

$$a - dp_t \over 1 + \sigma(1 - n_t^{INN}) = n_{t-1}^{INN} se^{bC} p_t + n_{t-1}^{IM} \mu n_{t-1}^{INN} se^{bC} p_t \quad (11)$$
Examples of dynamics

The industrial dynamics becomes irregular. Two examples of price and innovators fraction time series:

Time series of $p_t (\sigma = 90)$

Time series of $n_t^{INN} (\sigma = 120)$

Here $\beta = 5$, $C = 1$, $b = 0.5$, $\mu = 0.4$, $s = 2$, $a = 4$ and $d = 1$. 
Examples of dynamics

The industrial dynamics becomes irregular. Two examples of price and innovators fraction time series:

Time series of $p_t \ (\sigma = 90)$

<table>
<thead>
<tr>
<th>0</th>
<th>20</th>
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<tr>
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Time series of $n^{INN}_t \ (\sigma = 120)$

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Here $\beta = 5$, $C = 1$, $b = 0.5$, $\mu = 0.4$, $s = 2$, $a = 4$ and $d = 1$. 
The intensity of choice $\beta$

Bifurcation diagrams reveal irregular behaviour for large parameters regions.
Periodic orbits other than 2-cycles, as period 3, for instance.

Here $\sigma = 80$, $C = 1$, $b = 0.5$, $\mu = 0.4$, $s = 2$, $a = 4$ and $d = 1$. 
The generalized substitutability $\sigma$

As $\sigma$ gets larger the industry undergoes period 3 cycles, irregular behaviour, 4-cycles, 2-cycles, stable equilibrium.

Here $\beta = 5$, $C = 1$, $b = 0.5$, $\mu = 0.4$, $s = 2$, $a = 4$ and $d = 1$. 
The replicability factor $\mu$

Several orbits with different period.
Irregular behaviour for mid values of replicability.

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B.D. of $p_t$

Here $\sigma = 80$, $\beta = 5$, $C = 1$, $b = 0.5$, $s = 2$, $a = 4$ and $d = 1$. 
Basin of attraction for $\beta$ and $\sigma$

Red = stable equilibrium, Blue = 2-cycle, Green = 3-cycle, Yellow = 4-cycle, Cyan = 5-cycle, Purple = 6-cycle, Deep purple = 7-cycle, Orange = 8-cycle, Deep green = 9-cycle and Deep red = 10-cycle.

Non convergence is white, divergence is black.

Here $\mu = 0.4$, $C = 1$, $b = 0.5$, $s = 2$, $a = 4$ and $d = 1$. 
Final remarks

- When innovation also affects demand the industrial dynamics may become irregular.
- Several new periodic orbits appear other than 2-cycles and also a-periodic paths.
- In particular, period 3 cycles imply topological chaos (Li and Yorke, 1975).
- Loss of economic coherence with respect to the benchmark model, where innovation only affects supply.
- In general the model shows how the interplay between innovation and imitation produces complex dynamics.
- The interaction of factors as replicability and competition plays a major role.
Further research

- Model competition directly by making innovation costs and benefits dependent on firms’ fractions.

- Wait option of imitation strategy: only imitate successful innovations. Stochastic model.

- Increase the heterogeneity of firm population:
  - fast and slow imitation, asynchronous updating of routines: capture persistence.
  - heterogeneous expectations of price.

- Strategic innovation and imitation: bring Stackelberg model into the evolutionary discrete choice framework.